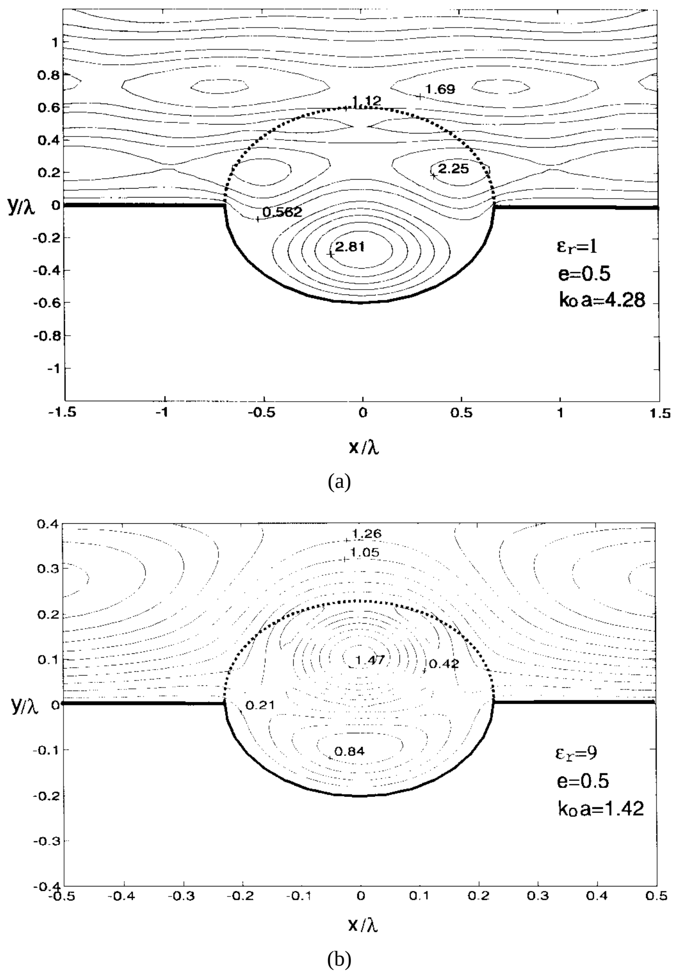

Fig. 5. Equiamplitude contour plots of $|E_z(\zeta, \eta)|$ near: (a) a hollow semielliptic channel ($k_0a = 4.28$, $\epsilon_r = 1$, $e = 0.5$, $\phi_i = 90°$) and (b) a dielectric-filled channel ($k_0a = 1.42$, $\epsilon_r = 9$, $e = 0.5$, $\phi_i = 90°$).

Fig. 5(b) for $\phi_i = 90°$, $\epsilon_r = 9$, $e = 0.5$, and $k_0a = 1.42$. The field pattern inside the channel is similar to the odd TM_{11} mode. From Fig. 5(a) and (b), it is seen that the boundary condition on the conducting plane is satisfied.

IV. Conclusion

An analytic-series solution based on the mode-matching method for hollow and dielectric-filled semielliptic channels is introduced in this paper. The validity and accuracy of the numerical results are examined by comparing the scattered-field pattern with those of the semicircular channels, which can be considered as the limiting geometry of the semielliptic one. The resonances in surface scattering with the semielliptic channels are seen to depend not only on the size of the channel and permittivity of the dielectric loading, but on the eccentricity. In addition, the scattered-field pattern depends very much on the eccentricity of the semielliptic channels.

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Efficient Parameter Computation of 2-D Multiconductor Interconnection Lines in Layered Media by Convergence Acceleration of Dielectric Green's Function via Padé Approximation

Ji Zheng and Zhengfan Li

***Abstract*—In this paper, a novel method is presented for calculation of the capacitance matrix of two-dimensional (2-D) interconnection lines embedded in layered dielectric media. In this method, Padé approximation is used to accelerate the convergence of Green's function, which leads to obvious improvements of computational efficiency for interconnect parameters. The obtained results show good agreement with those in previous publications.**

***Index Terms*—Capacitance, Green's function, interconnections.**

I. Introduction

Today, with increasing integration scale and clock frequency, the major limiting factors for further increasing the operating speed of integrated circuits (IC's) are interconnection delay and crosstalk, rather than the device switching speed. Parameter extraction for interconnects is a key step in the analysis of such delay and crosstalk

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J. Zheng was with the Department of Electronic Engineering, Shanghai Jiao Tong University, Shanghai 200052, China. He is now with the Department of Electrical and Computer Engineering, Oregon State University, Corvallis, OR 97331-3211 USA.

Z. Li is with the Department of Electronic Engineering, Shanghai Jiao Tong University, Shanghai 200052, China.

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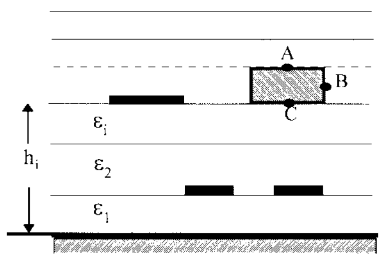

Fig. 1. The multiconductor interconnection lines embedded in multilayered dielectric media.

because its efficiency and generality dominate those of the whole analysis procedure. As is well known, multiconductor transmission lines embedded in multilayered dielectric media are the basic interconnection units in IC's and multichip modules (MCM's), and have been characterized with the distributed parameters $[C]$, $[L]$, $[R]$, and $[G]$ under quasi-TEM conditions.

The quasi-TEM methods of parameter computation can be roughly divided into two categories: the domain and boundary methods. Although the domain method can provide flexibility in dealing with complex objects, it must treat a great many segments, which needs much computer memory and central processing unit (CPU) time. Recently, some new methods in this category, such as the method of measured equation of invariance (MEI) [1], improved the computational efficiency through shrinking the boundary. For the second category of methods, i.e., the method of moments, the segments only exist on the surface of conductors so that the number of unknown variables is diminished. However, to solve the boundary integral equations involved in this kind of method, obtaining the Green's function is a crux step to produce the elements of the coefficient matrix. Generally, the Green's function in layered media is very complex and is always represented by infinite series with slow convergence, so the production of the matrix elements is very time consuming. An alternative method is the so-called total-charge Green's-function approach [2], [3], where the free-space Green's function is utilized. However, the additional segments on dielectric–dielectric interfaces must be included in this approach, so the computational efficiency is still not good. Some improved total-charge Green's-function methods, such as the multipole-acceleration method [4], have been presented for three-dimensional (3-D) capacitance computation.

Since the multilayered structures are indispensable for IC's, MCM's, and printed circuit-board (PCB) systems, in this paper, we deliver a novel method of parameter extraction for two-dimensional (2-D) interconnects embedded in planar multilayered dielectric media. The proposed method accelerates the convergence of the dielectric Green's function in infinite series via Padé approximation. It is common to give the solution by series expansion when dealing with complex boundary conditions in electromagnetics. However, previous series-acceleration approaches for electromagnetic-field computation [5]–[7] are focused on Green's functions related to wave equations, while the acceleration method for the static Green's function is seldom reported. The deduction of the dielectric Green's function is summarized in Section II of this paper, followed by the series acceleration via Padé approximation. Several examples of capacitance extraction for various kinds of 2-D interconnection lines are given in Section III to verify the efficiency of this method.

II. Principles

Consider the multiconductor interconnection lines embedded in multilayered dielectric media, shown in Fig. 1. The first step is to determine the Green's function in each layer for each line charge, such as A, B, and C on the conductor surfaces in the figure. The dominant equation is

$$\nabla^2 V_i(r'|r) = \delta(r'). \tag{1}$$

The Green's function is derived by considering a line charge on the dielectric–dielectric interface. Otherwise, if a line charge (such as A and B) is located within a layer, it would suffice to divide this layer into two new layers that are with the same dielectric property. The Laplace equation

$$\nabla^2 V_i(r'|r) = 0 \tag{2}$$

is solved in each dielectric layer, and the line-charge source is introduced by matching field quantity at the dielectric interfaces. For a specific line charge $\delta(x)\delta(y-h_i)$, the boundary conditions are

$$V_i = V_{i+1} \quad y = h_i, \qquad i = 1, 2, \cdots, N-1 \tag{3}$$

and

$$D_{yi+1} - D_{yi}|_{y=hi} = \delta(x). \tag{4}$$

The variable separation method is employed for solution of the Laplace equation. V_i is expressed as

$$V_i = \sum_{l=0}^{\infty} \cos(\alpha_l x)(A_{il}\exp(\alpha_l y) + B_{il}\exp(-\alpha_l y)) \tag{5}$$

where $\alpha_l = 2l\pi/L$ is a variable separation constant, and L is a presupposed virtual period in the x-direction and is set as four-to-five times the width of the area occupied by the interconnection lines. Keeping in mind the Fourier cosine expansion for $\delta(x)$

$$\delta(x) = \sum_{l=0}^{\infty} \frac{1}{\delta_l} \cdot \frac{2}{L} \cdot \cos(\alpha_l x) \tag{6}$$

where

$$\delta_l = \begin{cases} 2, & l = 0 \\ 1, & \text{otherwise} \end{cases}$$

substitution of (5) in (3) and (4) yields the solutions for the coefficients A_{il} and B_{il} in the Green's function. After the Green's function in layered media is determined, the capacitance matrix $[C]$ can be computed according to its definition by simply solving

$$\sum_{j=1}^{Nc} \int_{S_j} \sigma_j(r') \cdot G(r'|r)\, dr' = V(r) \tag{7}$$

through the general moment-method procedure with $V(r) = 1$ on each conductor surface while the potentials on the other conductor surfaces are zero, where N_c is the number of conductors, and σ_j is the unknown surface charge density on the conductor subsections. The kernel of the integral equation does not have any singularity even if r' and r are located on the same conductor subsection so that the integral equation can be easily solved.

Computational experiments show that the summation of the series of Green's functions in (5) takes up most of the CPU time. Considering that series expansion is a common approach for the solution of complex boundary-condition problems or eigenvalue problems in electromagnetic-field computation, it will be important to accelerate series-convergence speed. Several convergence acceleration methods based on "series transformation" have been reported, which transform series to a new one with faster convergence speed. The Kummer's transformation combined with Poisson summation formula, Shank's transform, and the well-known Aitken Δ^2 method are some examples [8]. However, acceleration approaches based on transformation require great effort to analyze the transformable aspect, which is characteristic for a specific series, and the attention of previous works is paid to the Green's functions for wave equations. Convergence acceleration of static Green's function is seldom reported in previous references, and numerical experiments show that the above transformation approaches do not contribute to the convergence acceleration of the static dielectric Green's function in layered media. In this paper, Padé approximation is used as a general approach for the series-convergence acceleration procedure and is irrespective of the functional form.

Padé approximation [9] is basically defined as a rational fraction approximation for a given real function $f(z)$

$$f(z) \approx P_m(z)/Q_n(z) \tag{8}$$

where

$$P_m(z) = \sum_{i=0}^{m} a_i z^i, \; a_i, \; z \in R$$

and

$$Q_n(z) = \sum_{i=0}^{n} b_i z^i, \; b_i, \; z \in R.$$

Let $f(z)$ be a power series, i.e., $f(z) = \sum_{i=0}^{\infty} c_i z^i$, and the (m, n)-order Padé approximation of $f(z)$ be denoted by $(m/n)_f = P_m(z)/Q_n(z)$; then, $P_m(z)$ and $Q_n(z)$ can be explicitly expressed by the following respective determinants:

$$P_m(z) = \det \begin{bmatrix} c_{m+1} & c_m & \cdots & c_{m-n+1} \\ c_{m+2} & c_{m+1} & \cdots & c_{m-n+2} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=0}^{m} c_i z^i & \sum_{i=0}^{m-1} c_i z^{i+1} & \cdots & \sum_{i=0}^{m-n} c_i z^{i+n} \end{bmatrix} \tag{9}$$

$$Q_n(z) = \det \begin{bmatrix} c_{m+1} & c_m & \cdots & c_{m-n+1} \\ c_{m+2} & c_{m+1} & \cdots & c_{m-n+2} \\ \vdots & \vdots & \cdots & \vdots \\ c_{m+n} & c_{m+n-1} & \cdots & c_m \\ 1 & z & \cdots & z^n \end{bmatrix}. \tag{10}$$

The essential point in the series' convergence acceleration is that if we evaluate the value of function $f(z)$ for $z = z_0$, its Padé approximation usually converges faster than $f(z)$ itself. This idea can be applied to the Green's-function expression (5). Let S_l be the lth item in the infinite series of (5) as follows:

$$S_l = \cos(\alpha_l x)(A_{il} \exp(\alpha_l y) + B_{il} \exp(-\alpha_l y))) \tag{11}$$

and assume a function $U(z) = \sum_{l=0}^{\infty} S_l z^l$, then $U(1)$ will give the infinite summation $\sum_{l=0}^{\infty} S_l$, which represents the solution of the Green's function. On the other hand, $(m/n)_u(1)$ is the Padé approximation of $U(1)$, so we can conclude that $(m/n)_u(1)$ gives an estimate of the original series instead of direct summation. It can be expected that $(m/n)_u(1)$ converges faster than $\sum_{l=0}^{\infty} S_l$. Although the general proof is impractical, we would prove that the Aitken Δ^2 series acceleration method is a special case of Padé approximation. Aitken Δ^2 is briefly summarized below. For sequence $s = \{s_n : n = 0, 1, 2, \cdots\}$, let

$$\begin{aligned} \Delta s_n &= s_{n+1} - s_n \\ \Delta^2 s_n &= \Delta(\Delta s_n) \\ &= s_{n+2} - 2s_{n+1} + s_n. \end{aligned}$$

A new sequence is defined as

$$T = \{T_n : n = 0, 1, 2, \cdots\}$$

where

$$T_n = s_n - \frac{(\Delta s_n)^2}{\Delta^2 s_n}.$$

It can be easily verified that the Aitken Δ^2 can give the optimal scheme for the series with a geometrical convergence rate. In fact, if $s_n = s - a\alpha^n, \; a \neq 0, \; |\alpha| < 1$, then

$$\begin{aligned} \Delta s_n &= a\alpha^n(1-\alpha) \\ \Delta^2 s_n &= -a\alpha^2(1-\alpha)^2 \\ T_n &= s - a\alpha^n + \frac{[a\alpha^n(1-\alpha)]^2}{a\alpha^n(1-\alpha)^2} \\ &= s. \end{aligned}$$

To elaborate the relationship between Padé approximation and the Aitken Δ^2 method, let S_n be the partial-summation sequence of a series $\{C_n\}$, i.e., $c_0 = s_0, c_{n+1} = \Delta s_n = s_{n+1} - s_n$; let $f(z) = \sum_{i=0}^{\infty} c_i z^i$, then $f(1) = s$ when s_n approaches s. On the other hand, we evaluate $f(1)$ by $f(z)$'s Padé approximation $(m/n)_f(1) s, m = 0, 1, 2, \cdots$, as follows:

$$(m/1)_f(1) = \frac{\det \begin{bmatrix} c_{m+1} & c_m \\ \sum_{i=0}^{m} c_i & \sum_{i=0}^{m-1} c_i \end{bmatrix}}{\det \begin{bmatrix} c_{m+1} & c_m \\ 1 & 1 \end{bmatrix}} = s_{m-1} - \frac{(\Delta s_{m-1})^2}{\Delta^2 s_{m-1}} = T_{m-1}.$$

Therefore, we conclude that Aitken Δ^2 is equivalent to Padé approximation $(m/1)_f(1)$. Since we have illustrated the validity for the series acceleration, we proceed with the evaluation of the Padé approximation. In practical application, we employ the Padé approximation with $n > 1$, and the determinants (9) and (10) are not adopted to compute the Padé approximation of the constructed function $U(z)$ because the computational complexity of solving determinants is of order $O(N^3)$, which is equivalent to that of solving linear equations. Assume $U(z) = \sum_{l=0}^{\infty} S_l z^l$, and

$$\begin{aligned} \varepsilon_{-1}^{j} &= 0 \\ \varepsilon_0^{j} &= \sum_{l=0}^{j} S_l z^l \\ \varepsilon_{k+1}^{j} &= \varepsilon_{k-1}^{j+1} + (\varepsilon_k^{j+1} - \varepsilon_k^{j})^{-1} \end{aligned}$$

then a long deduction will demonstrate that E_{2k}^{j} is the $(k+j, \; k)$-order Padé approximation of $U(z)$. The above recurrence formula is called the ε algorithm. The numerical tests given in Section III indicate that it usually only takes about ten items for the Padé approximation to converge.

III. Numerical Results

A general-purpose program based on the proposed method in this paper was developed to compute the distributed $[C]$ and $[L]$ parameters of the interconnection lines. The program deals with conductors with both finite and zero thickness. First, a pair of coupled interconnection lines on a dielectric slab over a conductor ground plane, shown in Fig. 2(a), is used to testify the convergence of Padé acceleration for the dielectric Green's function in layered dielectric media. The segmentation scheme is depicted in Fig. 2(b). We denote

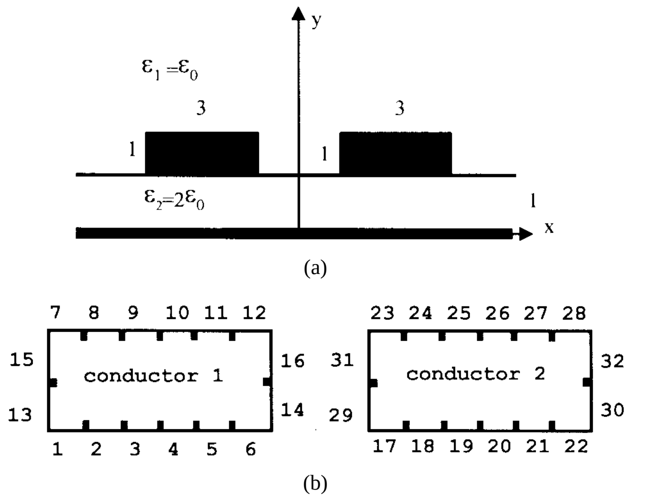

Fig. 2. (a) Coupled parallel interconnection lines. (b) The segmentation scheme for the structure in (a).

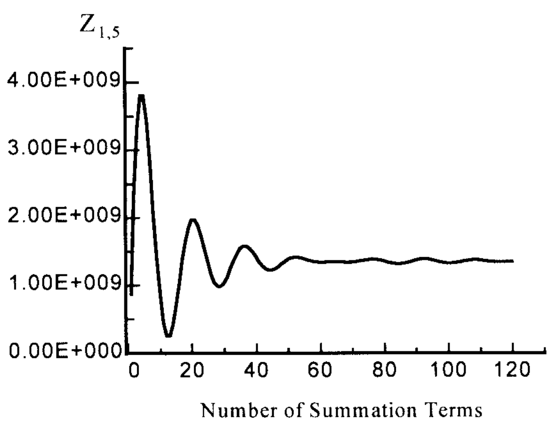

Fig. 3. The convergence curve of direct summation for $Z_{1,5}$.

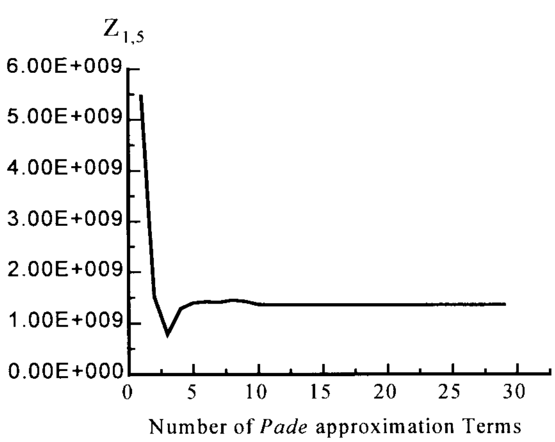

Fig. 4. The convergence cure of Padé approximation for $Z_{1,5}$.

the potential on the middle point of the ith segment due to the line charges along the segment j by $Z_{i,j}$. Fig. 3 shows the convergence curve of $Z_{1,5}$ as a function of the direct summation items. Fig. 4 shows the convergence curve of $Z_{1,5}$ accelerated by Padé approximation as a function of the approximation terms. It is obvious from these figures that the direct summation must take more than 100 terms to guarantee the convergence, while Padé approximation only

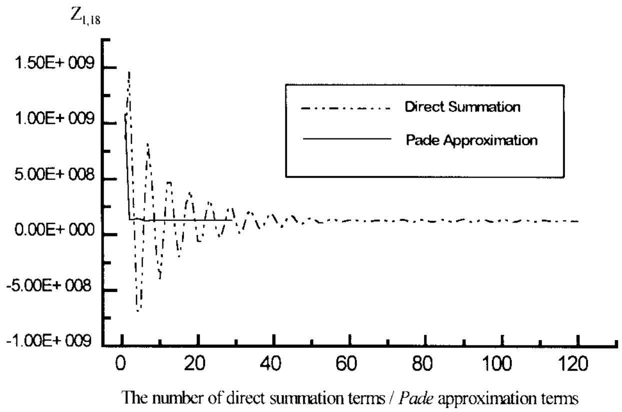

Fig. 5. The convergence comparison between direct summation and Padé approximation for $Z_{1,18}$.

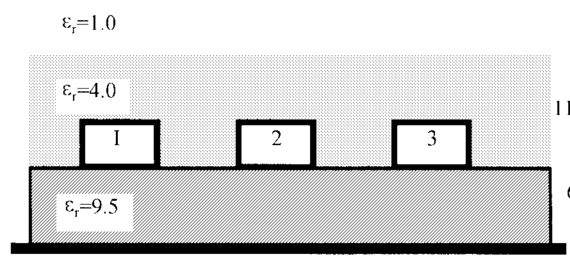

Fig. 6. Three-conductor interconnection line system.

TABLE I
COMPARISON OF RESULTS FOR EXAMPLE 1, FIG. 2

capacitance(F/m)	Our results	Reference[2]
C_{11}	0.9264366×10^{-10}	0.9165×10^{-10}
C_{12}	-0.8305213×10^{-11}	-0.8220×10^{-11}
C_{21}	-0.8305213×10^{-11}	-0.8220×10^{-11}
C_{22}	0.9264366×10^{-10}	0.9165×10^{-10}

needs approximately 15 terms to converge. Experimental data show that the difference between the results of 120-term direct summation and 15-term Padé approximation is negligible. When the distance between two segments increases, the direct summation curve appears to oscillate violently, as shown in Fig. 5, for $Z_{1,18}$, while it only takes several terms for Padé approximation to achieve excellent accuracy. The attenuative oscillation property makes the direct summation have to take at least 80 items to get enough accuracy. In general, the capacitance computation speed will be increased by more than three times when Padé approximation to the Green's function is employed.

Example 1

The capacitance matrix of the two-conductor structure is computed. A total of 32 subsections are used for conductor-to-dielectric interfaces. The results obtained are shown in Table I, which are in good agreement with the results of the total-charge Green's-function approach in [2]. The computational speed here is about three times faster than that of the total-charge Green's-function method.

Example 2

Three-conductor interconnection lines embedded in three dielectric layers in Fig. 6 is considered. The width of each conductor is eight

TABLE II
COMPARISON OF RESULTS FOR EXAMPLE 2, FIG. 6

capacitance(pF/m)	Our results	Reference[3]
C_{11}	266.459500	269.5200
C_{12}	-34.812590	-34.8680
C_{13}	-1.302291	-1.2567
C_{21}	-34.812590	-34.8680
C_{22}	274.743300	277.7500
C_{23}	-34.812590	-34.8680
C_{31}	-1.302291	-1.2567
C_{32}	-34.812590	-34.8680
C_{33}	266.459500	269.5200

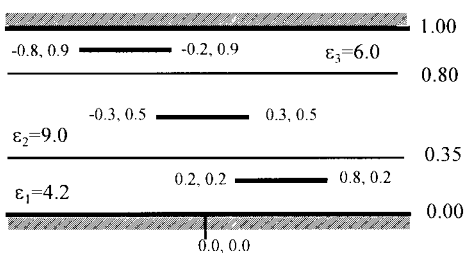

Fig. 7. Three infinitesimally thin interconnection lines (the coordinates are shown with the conductors in the figure).

and the height is six. The distance between adjacent conductors is ten. The left conductor is conductor one, the right conductor is conductor three, and the middle one is conductor two.

In this example, we treat a matrix much smaller in size than that used in the total charge free-space Green's function approach. Sixty-six subsections on the surfaces of the conductors and 255 additional subsections on the dielectric interfaces are used in [3]. In our proposed method, 60 subsections were used, and the results are given in Table II. The maximum difference between our results and those of [3] is less than 4% (for C_{13} and C_{31}). Since the distributed inductance matrix is determined as long as the capacitance matrix is obtained, we only present the results of $[C]$. In this example, less than ten items are used for Padé approximation, except for intensively coupled elements. In general, for such a type of interconnection structures, the new method is about five-to-six times faster than the total charge approaches.

Example 3

Three infinitesimally thin strip interconnection lines embedded in a three-layer dielectric between two ground planes is considered in Fig. 7. The coordinates of the conductors are also shown in the Fig. 7. The right-hand strip is conductor one, the center strip is conductor two, and the left-hand strip is conductor three. In [2], 132 subsections are used, while our approach needs only 18 subsections on the conductor surfaces. The computed results are listed in Table III.

TABLE III
COMPARISON OF RESULTS FOR EXAMPLE 3, FIG. 7

capacitance(pF/m)	Our results	Reference[2]
C_{11}	245.825800	245.9
C_{12}	-61.300010	-61.38
C_{13}	-0.5811030	-0.5737
C_{21}	-61.300010	-61.38
C_{22}	286.209300	286.5
C_{23}	-64.519580	-64.57
C_{31}	-0.5811030	-0.5737
C_{32}	-64.519580	-64.57
C_{33}	489.979700	490.0

IV. CONCLUSION

An accelerated method based on Padé approximation is presented for the first time to compute the capacitance matrices for 2-D interconnection lines in layered media. Since the dielectric Green's functions in the form of infinite series are effectively accelerated via Padé approximation, the efficiency of parameter computation by the integral-equation method is improved, as compared to the previous methods.

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